MAGNETIC RELAXATION AND THE ELECTROMAGNETIC RESPONSE PARAMETER

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Relaxation processes are important in the study of electromagnetic prospecting techniques. These processes can be the result of magnetic fields due to eddy currents in electrical conductors or due to displacement currents from dielectric processes or magnetic fields associated with direct magnetic loss phenomena. In the past, most attention has been focused on eddy-current losses since this is the dominant effect. Dielectric losses also may be of importance, particularly at high frequencies, where the conductivity is extremely low or where the dielectric properties become anomalously high. In general, however, little attention has been given to magnetic losses and the effect they might have on the electromagnetic response parameter. Magnetic minerals do have a measurable magnetic loss due to various mechanisms. This effect has been measured in a frequency range from 125 to 5000 hz for a variety of samples. In synthetic samples with varying amounts of magnetite dispersed in a nonmagnetic matrix the effect is linearly proportional to the magnetite content. The magnetic loss causes a distinct, but small, change in the electromagnetic response of a body. The effect can be detected, provided the peak frequency of magnetic loss is lower than the peak frequency of electrical loss. The most diagnostic feature is the reversed phase relationship from the electrical response (for constant geometries).

INTRODUCTION

Strangway (1970) has discussed possible occurrences of magnetic loss phenomena in electromagnetic material responses, and Colani and Aitken (1966) have discussed the response of magnetite in ancient hearthsites in terms of a magnetic relaxation loss mechanism. We will investigate here some of the magnetic effects theoretically expected in electromagnetic phenomena and then discuss some measurements of magnetic loss in minerals and possible applications.

The electromagnetic response of a sphere in free-space under the influence of a plane-wave source field has been considered by several authors (Wait, 1953; Grant and West, 1958; Ward, 1967; and others). For simplicity, we will use it in our calculations. The basic expression is of the form:

$$\mathbf{H} = H_0R^3(M - jN) \cdot \frac{(2x^2 - y^2 - z^2)\mathbf{i} + 3xy\mathbf{j} + 3xz\mathbf{k}}{r^5}, \tag{1}$$

where $\mathbf{H}$ is the secondary magnetic field measured at a distance r, H_0 is the source field, and R is the radius of the conducting sphere. M and N are the real and imaginary components of the electromagnetic response parameter containing all of the electric and magnetic parameters describing the material causing the response. $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the coordinate directions x, y, and z.

Following the notation of Ward (1967), we write:

$$(M-jN) = \frac{2\mu(\tan\gamma-\gamma)-\mu_0(\gamma-\tan\gamma+\gamma^2\tan\gamma)}{2\mu(\tan\gamma-\gamma)+2\mu_0(\gamma-\tan\gamma+\gamma^2\tan\gamma)}, \tag{2}$$

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where

μ is the permeability of the sphere,
μ_0 is the permeability of free space, and
$\gamma = kR$.
k is the propagation constant of the sphere and is given as:

$$k^2 = \mu\epsilon\omega^2 + j\mu\sigma\omega, \tag{3}$$

where

ϵ is the dielectric permittivity of the sphere in Farads/meter,
σ is the electrical conductivity of the sphere in mho/meter, and
ω is the radial frequency.

As shown by Miles et al (1957), Landau and Lifshitz (1960), and Fuller and Ward (1970), each of the quantities μ, σ, ϵ can be complex:

$$\begin{aligned} \mu &= \mu' - j\mu'' \\ \epsilon &= \epsilon' - j\epsilon'' \\ \sigma &= \sigma' - j\sigma''. \end{aligned} \tag{4}$$

In most earth materials, the conductivity term is assumed dominant (see Strangway, 1961). With the advent of lunar exploration and in a few terrestrial applications it has become important to consider ϵ, since the lack of moisture results in very low values of conductivity. Thus, both terms need to be considered in some applications.

As the loss is contained in the above complex quantities and is usually measured as a phase angle δ the power loss tangent can be written as (Miles et al, 1957)

$$\tan\delta = \frac{\operatorname{Im} k}{\operatorname{Re} k} = \tan(\delta'/2), \tag{5}$$

in which

$$\tan\delta' = \tan(\delta_E - \delta_M). \tag{6}$$

Here

$$\begin{aligned} \tan\delta_E &= \frac{\omega\epsilon'' + \sigma'}{\omega\epsilon'} = \tan\delta_D + \frac{\sigma'}{\omega\epsilon'} \\ \tan\delta_M &= \frac{\mu''}{\mu'}, \end{aligned} \tag{7}$$

in which σ' is the dc conductivity and all frequency dependent electrical constitutive relations are included in the complex permittivity.

Thus we see that the electric and magnetic loss components are independent and have opposite phases. The angle δ represents the phase relationship between the electric field E and the magnetic field H. δ_D is the phase between E and D, and δ_M is the phase between H and B, where

$$\begin{aligned} D &= \epsilon E \\ B &= \mu H. \end{aligned} \tag{8}$$

In (8) we have neglected magnetoelectric effects (O'Dell, 1971) as they are usually five or six orders of magnitude smaller than the dominant effects shown in the typical linear constitutive relations in (8). We also neglect nonlinear effects (Katsube et al, 1973), since the field strengths employed in our measurements and in exploration surveys are far below magnetic saturation levels.

For the purposes of evaluating the significance of magnetic loss with respect to eddy current loss in terrestrial prospecting systems, we have made the following simplifying assumptions [which are implicit in equation (2)]:

(1) displacement currents are negligible (neglect all dielectric contributions),

$$\gamma = kR \simeq R\sqrt{j\mu\sigma\omega} = \sqrt{j}\,\theta,$$

(2) the applied field is purely magnetic and uniform over the sphere

$$|Rk_{\text{medium}}| \ll 1.$$

We choose the first assumption in order to simplify the mathematics and to illustrate the relationship between the dominant eddy-current effect and the magnetic loss term without the additional complication of a third process. (In some cases, such as lunar studies and glacier and salt studies, the dielectric displacement current term probably should be explicitly considered.) The second assumption simplifies the effects of geometry. We may thus solve for M and N in equation (2) and plot them versus the normalized parameter θ as shown in Figure 1 (after Ward, 1967).

The numbers 1, 3, and 300 in Figure 1 refer to the values for the sphere permeability with respect to free space. Massive magnetite has an approximate value of 3 as shown by Werner (1945). Typical rocks have lower values depending on the magnetite content. It is apparent that

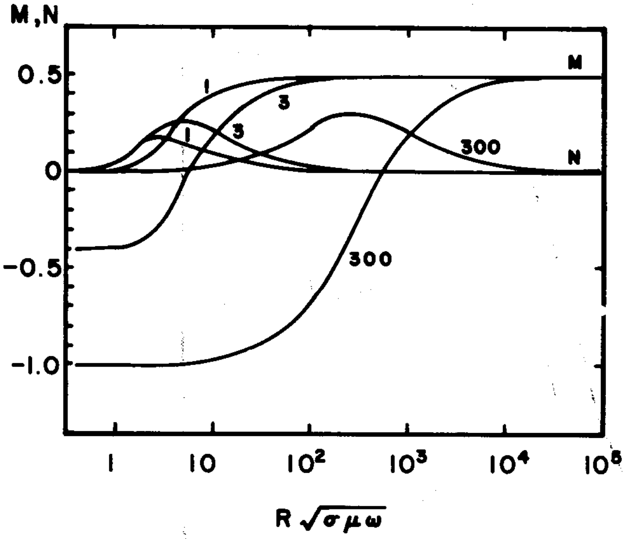

FIG. 1. *(M-jN)* versus $R\sqrt{\sigma\mu\omega}$ after Ward (1967), with the signs of M and N reversed to follow convention.

both M and N have the same sign when the sphere permeability is that of free space. When the permeability differs from the free space value of 1, the real part of the electromagnetic response M will change sign at frequencies greater than the frequency of the peak electrical loss. Peak electrical loss is defined as the peak of the quadrature component N. Ward (1967), Nabighian (1970, 1971), and others have reported solutions in which the permeability differs from free-space but is entirely real. We will now add complex, frequency-dependent permeabilities.

MAGNETIC LOSS

To consider the importance of magnetic loss in the electromagnetic response parameter, we first investigate the functional dependence of magnetic relaxation with frequency. Poole and Farach (1971) give an excellent account of the frequency spectra by comparing the magnetic relaxation processes to the more commonly known electrical relaxation phenomena. We will summarize here by considering only one possible spectrum, the Cole-Cole distribution (Cole and Cole, 1941), which fits our experimental data reasonably well.

With a Cole-Cole frequency distribution, the complex magnetic permeability is given as:

$$\mu = \mu' - j\mu'' = \mu_\infty + \frac{\mu_s - \mu_\infty}{1 + (j\omega\tau)^{1-\alpha}} \quad (9)$$

$$\mu_\infty = \lim_{\omega \to \infty} \mu';$$

where the static permeability

$$\mu_s = \lim_{\omega \to 0} \mu'.$$

τ = time constant such that when $\mu' = (\mu_s + \mu_\infty)/2, \omega\tau = 1$.

$1-\alpha$ = a distribution parameter in which the limits are (Fuoss and Kirkwood, 1941):

$1-\alpha = 1$, a single Debye-like relaxation mechanism; and

$1-\alpha = 0$, an infinitely broad distribution of relaxation times.

The $1-\alpha=0$ limit is physically nonrealizable as it occurs for $\mu''=0$. However, approaching this limit produces loss tangents nearly independent of frequency with a time domain response proportional to the logarithm of time.

With such a frequency-dependent, complex permeability, we will have to separate the permeability and frequency from the normalizing parameter θ in Figure 1. $(M-jN)$ is plotted in Figure 2 for varying $R\sqrt{\sigma}$ and no magnetic loss. For fixed R, it may be seen that increasing the conductivity lowers the peak relaxation frequency, indicating the change in depth penetration of the eddy-current.

Adding the complex permeability in the form of a Cole-Cole frequency distributed magnetic relaxation loss, we see that $(M-jN)$ is modified giving the electromagnetic response parameter shown in Figures 3 and 4. The differing phase of the electrical and magnetic components is shown by the reversed sign in N at the peak frequencies of the magnetic and eddy current loss respectively. Figure 3 shows the effect when $\tau_M \leq \tau_E$ [τ_M = time constant of magnetic relaxation from equation (9), τ_E = time constant of electrical relaxation, defined as in Figure 2, the reciprocal of the frequency for which the quadrature component N peaks], and the eddy-current (surface dependent) electrical response completely dominates the (volume dependent) magnetic loss. For $\tau_M \gg \tau_E$ we see that the magnetic effect can be detected by a change in sign of N at the peak frequency of magnetic loss $\sim\tau_M^{-1}$. This leads to the following cases:

(1) Eddy current loss only $\mu'=1$, $\mu''=0$

$$\left.\begin{matrix} M > 0 \\ N > 0 \end{matrix}\right\} \quad \text{all } \omega,$$

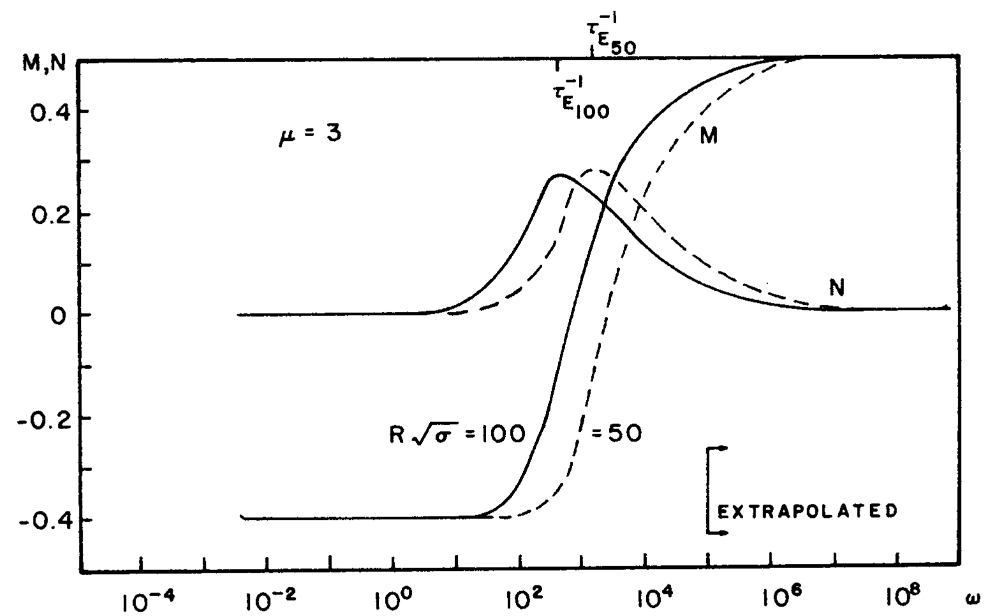

FIG. 2. $(M\text{-}jN)$ versus frequency for two values of $R\sqrt{\sigma}$ at fixed sphere permeability of 3. (In this and succeeding figures, the values above 10^5 hz are extrapolated from the behavior of Figure 1 as the assumptions chosen do not allow explicit calculation.) $\mu'' = 0$.

(2) Eddy current loss plus ferromagnetism without loss

$$\mu' > 1, \qquad \mu'' = 0$$

$$\left.\begin{array}{l} M > 0 \\ N > 0 \end{array}\right\} \qquad \omega > \tau_E^{-1}$$

$$\left.\begin{array}{l} M < 0 \\ N > 0 \end{array}\right\} \qquad \omega < \tau_E^{-1},$$

(3) Eddy current loss and ferromagnetic loss

$$\mu' > 1, \qquad \mu'' > 0,$$

(a)

$$\tau_M \leq \tau_E$$

same as (2) above,

(b)

$$\tau_M \gg \tau_E$$

$$\left.\begin{array}{l} M > 0 \\ N > 0 \end{array}\right\} \qquad \omega > \tau_E^{-1}$$

$$\left.\begin{array}{l} M < 0 \\ N > 0 \end{array}\right\} \qquad \omega < \tau_E^{-1}$$

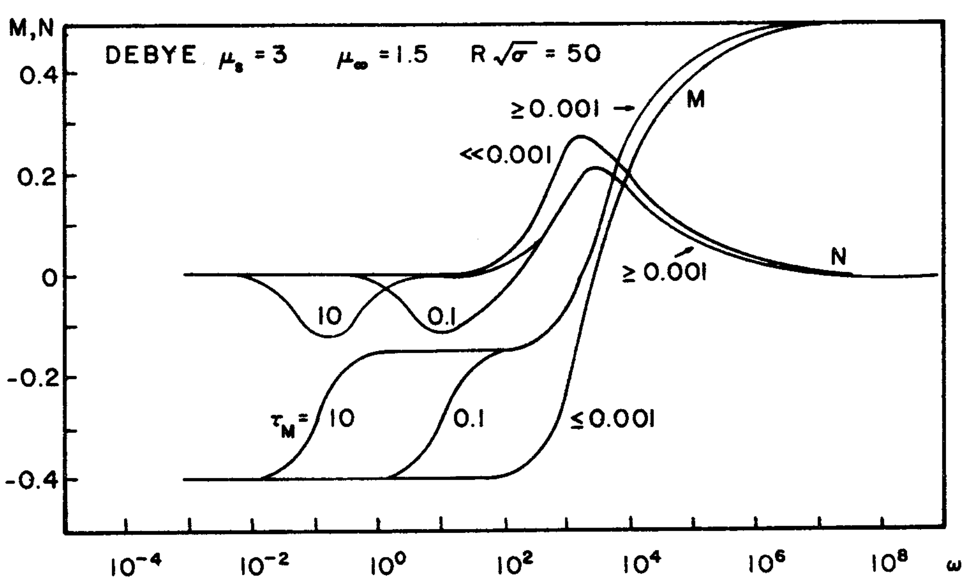

FIG. 3. $(M\text{-}jN)$ versus frequency including Debye relaxation magnetic loss showing the effect of the magnetic time constant variation.

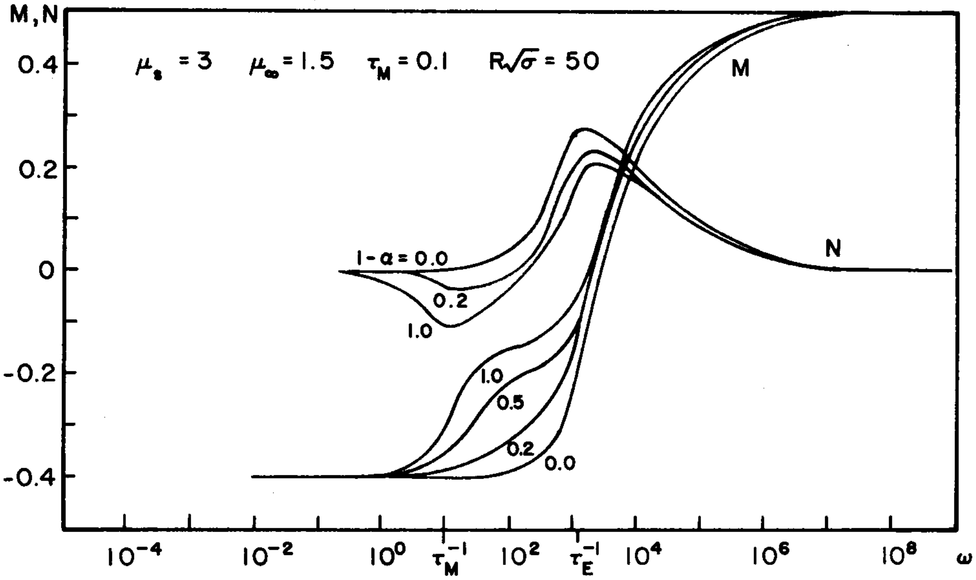

FIG. 4. *(M-jN)* versus frequency for varying magnetic relaxation Cole-Cole frequency distributions. The plot of N for $1-\alpha=0.5$ has been deleted for clarity as it is very close to that for $1-\alpha=0.2$.

$$\left.\begin{array}{l} M < 0 \\ N < 0 \end{array}\right\} \qquad \omega \approx \tau_M^{-1}.$$

These ranges are only qualitative as the sign changes in M and N do not occur exactly at τ^{-1} but shift slightly depending upon the exact values of conductivity and permeability. Also, in case 3b, if $\tau_M > \tau_E$ and the magnetic loss is small, N may remain positive and only dip slightly instead of changing sign.

Figure 4 shows the effect of the distribution parameter $1-\alpha$. The maximum possible loss tangent due to magnetic loss as shown in Figures 3 and 4 is 0.5 with $1-\alpha=1.0$.

Galt (1952) has measured a maximum loss tangent of 0.46 at 20 khz using very carefully prepared, single crystal magnetite. The maximum loss tangent we have observed for naturally occurring, polycrystalline magnetite is 0.05 (see below). Thus, the actual effect is one-tenth or less the size of that shown in Figures 3 and 4. Colani and Aitken (1966) have reported the use of magnetic loss techniques to detect ancient hearth-sites by their magnetic mineral content.

EXPERIMENTAL OBSERVATIONS

Measurement techniques for magnetic relaxation phenomena may be found in Galt (1952), Tomono (1952), Miles et al (1957), ASTM (1964), Olhoeft (1972), and Mullins and Tite (1973). Becker (1951), Street and Wooley (1949), Williams et al (1950), and Pry and Bean (1958) discuss theoretical and experimental results for audio-frequency magnetic losses. Becker (1959), Bozorth (1951), Ratheneau (1959), Chikazumi (1964), Sparks (1964), and Poole and Farach (1971) review the various types of magnetic loss mechanisms from very low frequencies (periods in years) to very high frequencies ($>10^9$ hz). Gevers (1945) and Shuey and Johnson (1973) are excellent sources for general theory of relaxation.

Nagata (1961), Strangway (1967), and others have discussed the general magnetic properties of minerals. Neel (1949) developed the theory of ferrimagnetism (Brown, 1965; Smart, 1966), and the theory of magnetic viscosity having to do with wide relaxation distributions in fine grain, superparamagnetic interactions (LeBorgne, 1960; Gose and Carnes, 1973). Standley (1972) has an excellent collection of magnetic properties including audio-frequency magnetic loss for oxide magnetic materials.

We have used an experimental arrangement similar to that used by Colani and Aitken (1966). Figure 5 schematically shows the arrangement. The procedure is as follows. A balanced pair of receiving coils is placed within a long cylindrical source solenoid. The coils are connected in parallel so that a current pulse in the solenoid produces equal and opposite (canceling) voltages in the pair of coils. A sample is placed into one of the coils of the balanced pair to unbalance the assembly so that the current pulse in the solenoid when

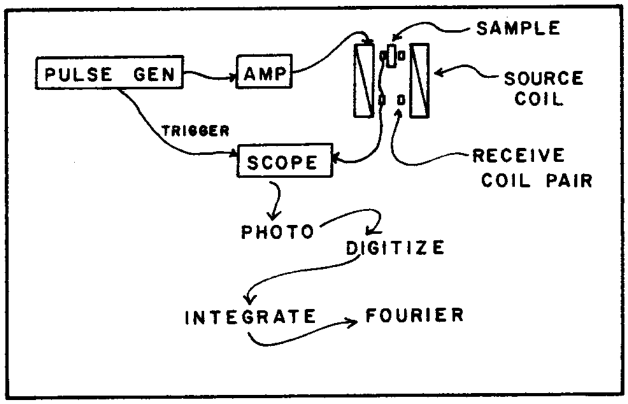

FIG. 5. Experimental procedure and data processing.

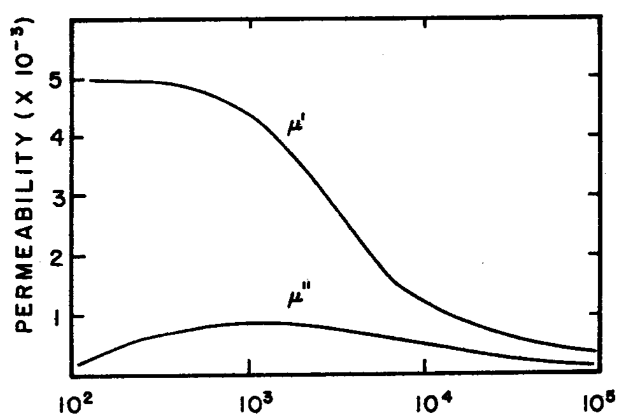

FIG. 6. Galt (1952) single crystal magnetite permeability spectrum.

magnetizing the sample and the magnetization decay when the pulse is turned off will produce a voltage output at the balanced-coil-pair output proportional to the time rate of change of the sample magnetization. This voltage is then integrated and transformed into the frequency domain with a Fourier series expansion (Hildebrand, 1956) to determine the real and imaginary components of the magnetic behavior. These can be combined and written as a loss tangent:

$$\tan \delta_M = \frac{\mathrm{Im}\,\mu}{\mathrm{Re}\,\mu} = \frac{\xi(\omega\tau)^{1-\alpha} \sin (1-\alpha)\pi/2}{1+\xi+(2+\xi)(\omega\tau)^{1-\alpha}\cos(1-\alpha)\pi/2+(\omega\tau)^{2(1-\alpha)}}, \tag{10}$$

where

$$\xi = \frac{\mu_s - \mu_\infty}{\mu_\infty}.$$

The primary source solenoid supplied fields up to 5 gauss, though typically measurements were performed using 1 gauss in order to remain in the linear region of the *B-H* hysteresis curve. Primary pulse length was 2.0 msec with an equivalent quiet period for measurements (alternate pulses were of opposite polarity). The typical transfoi med output was usable over a frequency range of 125–4000 hz.

Figure 6 displays Galt's (1952) data for single crystal magnetite. Figure 7 shows Galt's data replotted as a loss tangent in comparison with our measured magnetic loss tangent for naturally occurring polycrystalline magnetite. Galt's loss tangent has a slope indicating a distribution parameter of $1-\alpha=0.37$, a relatively broad distribution of relaxation times. Our measured results approach those reported by Galt at low frequencies, but differ widely at the higher frequencies. We attribute this difference to the probable disparity in number of domain walls and their freedom of movement between single crystal and polycrystalline states. Indeed, Galt attributes the peak in the loss tangent at 20 khz to domain wall movement. The error bars at the low frequencies in our data are the result of the coil response.

Figure 8 displays the variation of the loss tangent with changing quantities of magnetite in artificial samples constructed of Secar cement with given weight percentages of magnetite. The high-frequency error bars are caused by uncertainties introduced by the use of a high-frequency filter to reduce noise. Figure 9 shows a plot of ξ versus percent magnetite as measured from the natural sample (99 percent) and the artificial samples. Table 1 lists the various parameters found to characterize the magnetic response. For each parameter, the range of values shown corresponds to the error bars indicated. The errors are primarily caused by the restricted coil response at low frequencies.

The transformed relaxations yielded distributions whose loss tangents all peak in the region below 550 hz. Multiple coil sets with differing resonance properties, differing magnetization pulse lengths (50 μsec to 8 msec) and direct frequency-domain measurements (Miles et al, 1957) were used to verify that the observed

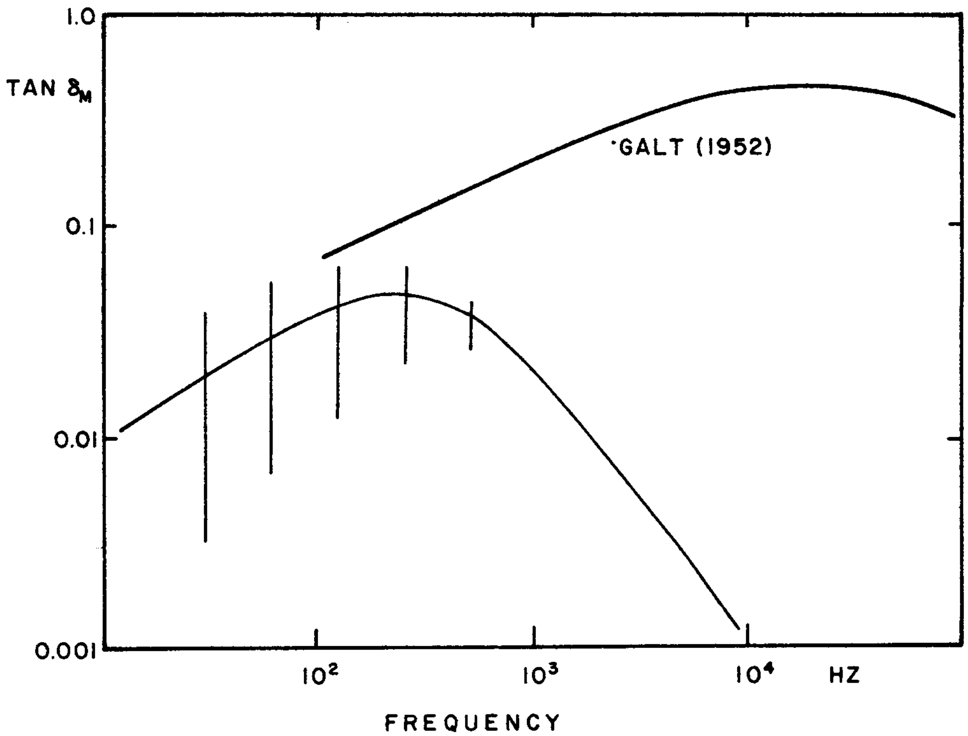

FIG. 7. Figure 6 replotted as a loss tangent with our measured loss tangent of naturally occurring polycrystalline magnetite.

relaxations were due to the samples and not an artifact of the measurement system.

In addition to ferrimagnetic mineral samples, several nonmagnetic, highly conductive samples were measured to observe the amount of eddy-current contribution in the system. No response was observed for monocrystalline samples of pyrite, chalcopyrite, or an aluminum rod. A piece of copper rod, however, showed the response pictured in Figure 10. This gave a transformed

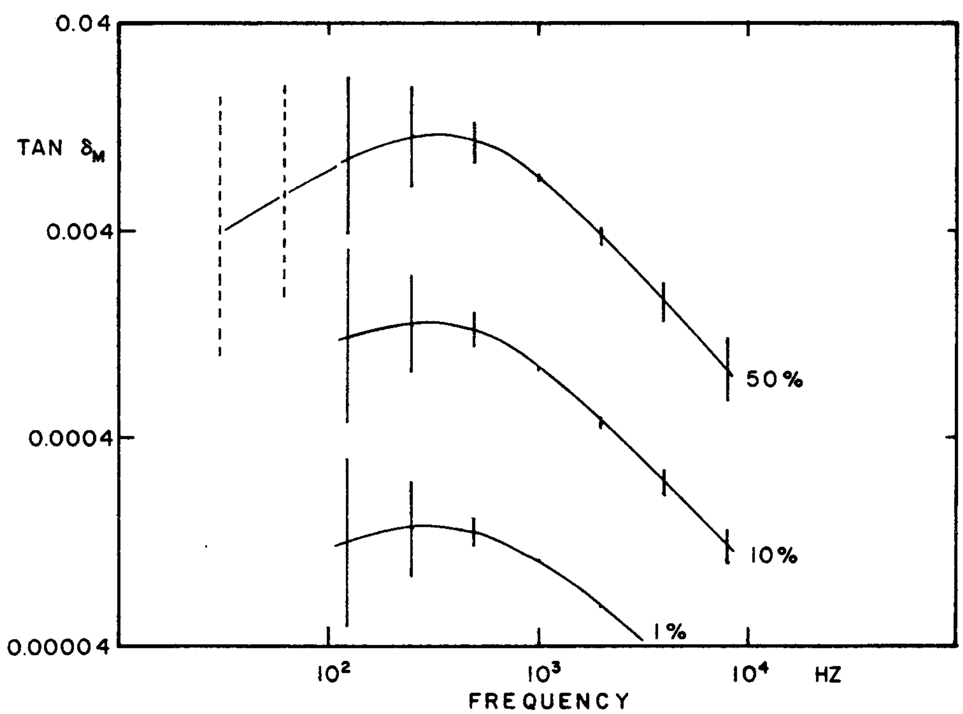

FIG. 8. Several loss tangent spectra for artificial samples with varying magnetite content; the weight percentages of magnetite are indicated.

result which exhibited the reverse phase from that exhibited by the ferrimagnetic samples confirming that it was an eddy current response. The amount of eddy current response for the minerals in Table 1 was typically at or below the noise level of the system which was near the equivalent signal of 1 percent magnetite.

DISCUSSION

We have shown that magnetic relaxation loss is theoretically an observable modification to the electromagnetic response ($M-jN$) of a sphere. We have further shown that magnetic loss is measurable for ferrimagnetic minerals in the laboratory. From the above laboratory evidence and the theoretical calculations it may be seen that only in case 3b above would ferrimagnetic mineral relaxation loss be observable. This implies that magnetite finely dispersed in a highly resistive matrix (volcanics for example) should give a measurable response. In such resistive areas where the eddy-current loss is small, the magnetic loss could therefore be important in creating a background effect.

If a survey were to be taken with narrow-band

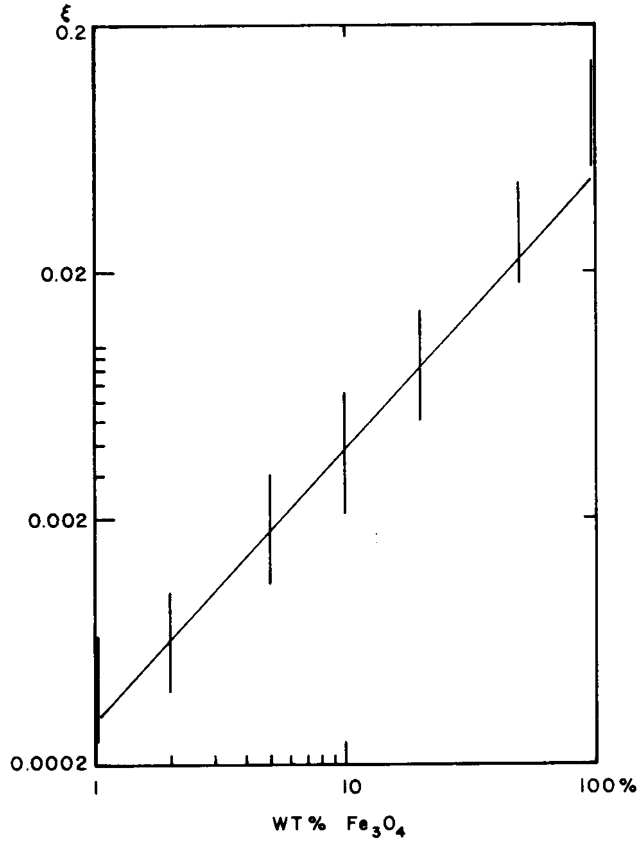

FIG. 9. ξ versus magnetite content from loss tangent data.

Table 1. Summary of Magnetic Relaxation Measurements

25°C	$\omega < \sim 2\pi$ khz $\omega > \sim 2\pi$ khz		$1-\alpha=0.85\to\to 1.0$ $1-\alpha=0.99\to\to 1.0$
Sample	τ_M ($\times 10^{-6}$ sec)	μ_s	ξ
Massive magnetite Ishpeming	362–853	1.5	0.053 –0.149
Niccolite Ontario	262–301	—	0.0038–0.011
Pyrrhotite Ontario	322–1624	1.36	0.015 –0.041
Massive sulfide Bathurst, N.B.	282–1778	—	0.0037–0.011
Hematite Belo Horizonte	375–1276	—	0.0013–0.0048
Artificial samples: [200 mesh magnetite in the indicated percentages (by weight) in Secar cement]			
Secar 1% Fe_3O_4	292–1061	—	0.00025–0.00067
2%	294–1990	—	0.004 –0.0010
5%	299–1763	—	0.011 –0.0030
10%	309–1774	1.035	0.0021 –0.0065
20%	293–2466	1.07	0.005 –0.014
50%	311–1356	1.22	0.018 –0.046

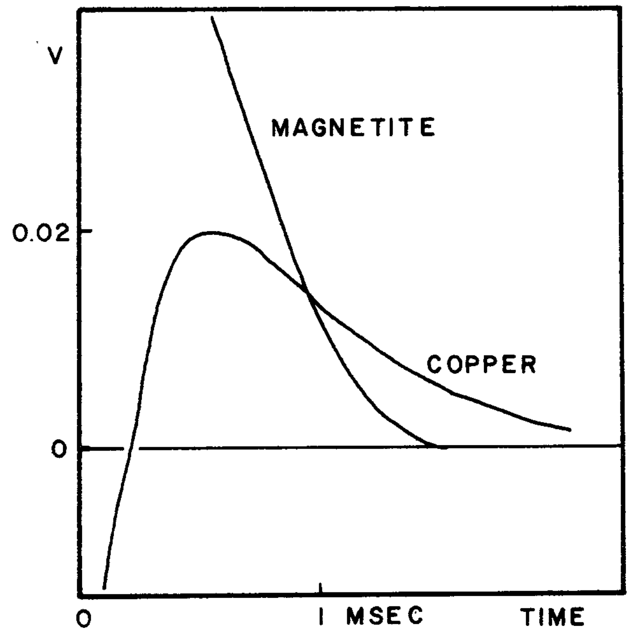

FIG. 10. Decay curves for magnetite and an equivalently sized copper rod showing the difference between magnetic relaxation and eddy current relaxation as observed at the output of the balanced coil pair.

equipment in a sufficiently resistive area with high ferrimagnetic mineral content (well dispersed), the magnetic relaxation response could be mistaken for the eddy current response with the resultant problems of interpretation. This will become particularly true as newer, more sensitive electromagnetic survey systems become available. However, a properly calibrated system measuring both amplitude and phase should be able to distinguish the magnetic response on the basis of the reversed phase information from that expected for eddy-current response.

CONCLUSIONS

We conclude that magnetic loss can be important in the study of the electromagnetic response of rocks and minerals. The effect found experimentally peaks for magnetite in the audio-frequency electromagnetic region near 350–1000 hz independent of geometry and has the opposite phase to the expected eddy-current response, which has a peak frequency, dependent upon both conductivity and geometry. When compared to the normal effects of eddy-current response over conductive areas, the effect can be expected to be quite small and in general not significant. In highly resistive areas, however, the eddy-current response in the audio-frequency electromagnetic region is small and the magnetic loss could well be important as a background effect. With the advent of more sophisticated electromagnetic systems, this effect could become relatively more important and permit mapping of magnetite content with an active system. As a background effect it could also represent an additional source of noise to further limit electromagnetic exploration.

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REFERENCES

American Society for Testing and Materials, 1964, ASTM standards relating to magnetic properties: Philadelphia.

Becker, R., 1951, La dynamique de la paroi de Bloch et la permeabilities en haute frequence: J. Phys. Rad., v. 12, p. 331.

——— 1959, Metallurgical structure and magnetic properties, *in* Magnetic properties of metals and alloys: Am. Soc. for Metals, Cleveland, p. 68.

le Borgne, E., 1960, Etude experimentale du trainage magnetique dans le cas d'un ensemble grains magnetique tres fins disperses dans une substance non magnetique: Ann. des Geophysiques, v. 16, p. 445.

Bozorth, R. M., 1951, Ferromagnetism: New York, Van Nostrand.

Brown, W. F., Jr., 1965, The theory of thermal and imperfection fluctuations in ferromagnetic solids, *in* Fluctuation phenomena in solids: ed. R. E. Burgess, New York, Academic Press, p. 37–79.

Chikazumi, S., 1964, Physics of magnetism: New York, John Wiley and Sons.

Colani, C., and Aitken, M. J., 1966, Utilization of magnetic viscosity effects in soils for archaeological prospection: Nature, v. 212, p. 1446.

Cole, K. S., and Cole, R. H., 1941, Dispersion and adsorption in dielectrics: J. Chem. Phys., v. 9, p. 341.

Fuoss, R. M., and Kirkwood, J. G., 1941, Electrical properties of solids: VIII, Am. Chem. Soc. J., v. 63, p. 385–394.

Fuller, B. D., and Ward, S. H., 1970, Linear system description of the electrical parameters of rocks: IEEE Trans. Geosci. Electron., Ge-8, no. 1.

Galt, J. K., 1952, Motion of a ferromagnetic domain wall in Fe304: Phys. Rev. 85, p. 664.

Gevers, M., 1945, The relation between the power factors and the temperature coefficient of the dielectric constant of solid dielectrics: Philips Res. Rep. 1, p. 197, 279, 298, 362, 447 (5 parts).

Gose, W. A., and Carnes, J., 1973, The time dependent magnetization of fine-grained iron in lunar breccias: Earth and Plan. Sci. Lett.

Grant, F. S., and West, F. G., 1958, Interpretation theory in geophysics: New York, McGraw-Hill Book Co., Inc.

Hildebrand, F. B., 1956, Introduction to numerical analysis: New York, McGraw-Hill Book Co., Inc., p. 373–377.

Katsube, T. J., Ahrens, R. H., and Collett, L. S., 1973, Electrical nonlinear phenomena in rocks: Geophysics, v. 38, p. 106–124.

Landau, L. D., and Lifshitz, E. M., 1960, Electrodynamics of continuous media, New York, Pergamon Press, p. 247–262.

Miles, P. A., Westphal, W. B., and von Hippel, A., 1957, Dielectric spectroscopy of ferromagnetic semiconductors: Rev. Mod. Phys., v. 29, p. 279.

Mullins, C. E., and Tite, M. S., 1973, Magnetic viscosity, quadrature susceptibility, and frequency dependence in single-domain assemblies of magnetite and maghemite: J. Geophys. Res., v. 78, p. 804.

Nabighian, N. M., 1970, Quasistatic transient response of a conducting, permeable sphere in a dipolar field: Geophysics, v. 35, p. 303.

——— 1971, Quasistatic transient response of a conducting, permeable, two-layer sphere in a dipolar field: Geophysics, v. 36, p. 25.

Nagata, T., 1961, Rock magnetism: Tokyo, Maruzen.

Neel, L., 1949, Theorie du trainage magnetique des ferromagnetiques en grains fins avec applications aux terres cuites: Ann. de Geophys., v. 5, p. 99.

O'Dell, T. H., 1970, The electrodynamics of magneto-electric media: Amsterdam, North-Holland.

Olhoeft, G. R., 1972, Time dependent magnetization and magnetic loss tangents: M.S. thesis, M.I.T.

Poole, C. P., Jr., and Farach, H. A., 1971, Relaxation in magnetic resonance: New York, Academic Press.

Pry, R. H., and Bean, C. P., 1958, Calculation of the energy loss in magnetic sheet materials using a domain model: J. Appl. Phys., v. 29, p. 532.

Ratheneau, G. W., 1959, Time effects in magnetization, *in* Magnetic properties of metals and alloys: Am. Soc. Metals, Cleveland, p. 168.

Shuey, R. T., and Johnson, M., 1973, On the phenomenology of electrical relaxation in rocks: Geophysics, v. 38, p. 37–48.

Smart, J. S., 1966, Effective field theories of magnetism: Philadelphia, M. B. Saunders Co.

Sparks, M., 1964, Ferromagnetic relaxation theory: New York, McGraw-Hill, Book Co., Inc.

Standley, K. J., 1972, Oxide magnetic materials: London, Oxford Press.

Strangway, D. W., 1961, Electromagnetic scale modeling, *in* Methods and techniques in geophysics: ed. S. K. Runcorn, v. 2, New York, Interscience, p. 1.

——— 1967, Mineral magnetism, *in* Mining geophysics, Vol. II: Tulsa, SEG, p. 437.

——— 1970, Possible electrical and magnetic properties of near-surface lunar materials, *in* Electromagnetic exploration of the moon: ed. W. Linlow, Mono Book Corp.

Street, R., and Woolley, J. C., 1949, A study of magnetic viscosity: Proc. Roy. Soc. (London), v. A62, p. 562.

Tomono, Y., 1952, Magnetic after-effect of cold rolled iron: J. Phys. Soc. Japan, v. 7, p. 174.

Wait, J. R., 1953, A conducting permeable sphere in the presence of a coil carrying an oscillating current: Can. J. Phys., v. 31, p. 670.

Ward, S. H., 1967, Electromagnetic theory for geophysical application, *in* Mining geophysics, Vol. II: Tulsa, SEG, p. 10.

Werner, S., 1945, Determinations of the magnetic susceptibility in ores from Swedish iron ore deposits: Swedish Geol. Survey, v. 39, p. 5.

Williams, H. J., Shockley, W., and Kittel, C., 1950, Studies of the propagation velocity of a ferromagnetic domain boundary: Phys. Rev. 80, p. 1090.

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